PRABHU DAYAL PUBLIC SCHOOL

SHALIMAR BAGH,DELHI-88

ASSIGNMENTS OF APPLICATION OF DERIVATIVES.

.Q-1 find the equation of the tangent to the curve y=$\frac{x-7}{(x-2)(x-3)}$ at the point where it cut the x axis. Also find the equation of tangent at that point.

Q-2 Find the equation of the tangent to the curve y=x^2-2x+7, which is (i) Parallel to the line y=2x+9 , (ii)perpendicular to the line 5y-15x=13.

Q-3 Find the equation of normal to the curve $y=x^3+2x+6, which\ are\ parallel\ to\ x+14y+4=0$

Q-4 Prove that curves $x=y^2, xy=k, cut\ at\ right\ angles\ if 8k^2=1$.

Q-5 Find the equation of normal to the curve $x^2=4y$ $passing\ through\ P(1,2)$.

Q-6 Find the $eq^n\ of\ tangent\ to\ the\ curve\ x=asin^3t, y=bcos^3t\ \ at\ \ t=\frac{\pi}{2}$

Q-7 Find the points on the curves y=x^3at which slope of tangent is equal to the y coordinate.

Q-8 If the tangent to the curve y=x^3 +ax +b at (1,-6) is parallel to the line x-y+5=0, and then find the values of a&b.

Q-9 Find the equation of the tangent to the curve y=$\sqrt{3x-2}$ which is parallel to the line 4x-2y+5=0,

Q-10 Prove that $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ touches the straight line x/a+ y/b=2 for all natural number n,at the point (a,b) **PAWAN**

Q-11 Show that the curves xy=a^2 , and $x^2+y^2=2$ touch each other.

Q-12 for the curve y=$4x^3-2x^5$,find all the point at which the tangent passes through the origin.

Q-13 Show that the line x/a +y/b touches the curve y= b$e^{\frac{-x}{a}}$ at the point where the curve cuts y-axis.

Q-14 Find the equation of the normal to the curve x^2 =4y, which passes through the point (1,2).also find the equation of corresponding tangent.

Q-15 the curve y=ax^3 +b x^2 +cx +5 touches the x axis at P (-2, 0) & cuts the y axis at the point Q where its gradient is 3. Find the value of a, b, c.

Q-16 Find the angle of intersection between the curves y^2 =4x, x^2 =4y.

Q-17 Prove that all the normal to the curve x= k cost+kt sint,y=k sint-kt cost are at a distance k from the origin.

Q-18 Find the equation of all the tangents to the curve y=cos(x+y),$-2\pi \leq x \leq 2\pi$. That are parallel to the linex+2y=0.

Q-19 Find the points on the curve $9y^2 = x^3$, where the normal to the curves makes equal intercepts with the coordinate axes.

Q-20 At what points will the tangent to the curve $x^2 + y^2$ -2x-3=0,be parallel to x-axis. also find the corresponding tangents.

Q-21 Approximate $f(5.001), where\ f(x) = x^3 - 7x^2 + 15,$ **PAWAN**

Q-22 Approximate $(3.968)^{3}/_{2}$,Q-23 Approximate $\sqrt{o.o37}$

Q-24 The radius of a sphere is 3 cm,if an error of o.o3is made in measuring the radius of the sphere, find the error and percentage error in measuring the surface area.

Q-25 Two equal sides of an isosceles triangle with fixed base 10 cm are decreasing at the rate of 3cm/sec.How fast is the area decreasing when two equal sides are equal to the base.

Q-26 Sand is pouring from a pipe at the rate of $12cm^3/\sec$.The falling sand forms a cone on the ground in such a way that the height of the cone is always one sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm.

Q-27 A particle moves along the curve $6y = x^3 + 2, find$ the points on the curve at which the y coordinate is changing 8 times as fast as the x coordinate.

Q-28 Water is dripping out at the uniform rate of 1 cm^3/sec through at a tiny whole at the vertex of a conical vessel whose axis is vertical.when the slant height of water is in the vessel is4 cm.find the rate of change of (a) the slant height of water.(b) the area of water surface, When the vertical angle is 60^0 .

Q-29 A man of 1.6 m tall walks at a rate of 0.5 m/sec away from a lamp post, 8 m high. Find the rate at which his shadow is increasing when he is 10 m away from the lamp post.

Q-30 It is given that for the function f(x)=x^3+bx^2+ax+5 0n[1,3],Role's theorem holds with c=2+$\frac{1}{\sqrt{3}}$ find the values of a&b.

Q-31 Verify the Rolle's for f(x)=(x-2)(x-3)(x-4) on x∈[2,4]

Q-32 Verify the Role's for f(x)=sinx+cosx-1, on x∈[0,$\pi/2$]

Q-33 Using theorem of Rolle's, find the point in (-2,2) on the curve y= $\boldsymbol{x^2}$ where the tangent parallel to x axis.

Q-34 Verify the Lagrange's mean value theorem for f(x)=(x-1)(x-2)(x-3) on x∈[0,4]

Q-35 Verify the Lagrange's mean value theorem for f(x)=six-sin2x on x∈[0,π]

Q-36 Using L.M.V. theorem to determine a point on the curve y=$\sqrt{x^2-4}$ defined in the interval [2,4].where the tangent is parallel to the chord joining the and points to the curve. **PAWAN**

Q-37 Verify the Role's theorem for f(x) =e^xcosx, on x ∈ $[\pi/2, \pi/2]$

Q-38 Find the intervals in which $f(x) = sinx + cosx$,$0 \leq x \leq 2\pi$ is strictly increasing or decreasing.

Q-39 Find the increasing and decreasing intervals for

(i)$f(x) = -2x^3 - 9x^2 - 12x + 1$ (ii)$f(x) = x^3 + \frac{1}{x^3}$

(iii)$f(x) = (x+1)^3(x-3)^3$ (iv) f(x)=log(1+x) - $\frac{2x}{2+x}$

(v) f(x)= (x+2)e^{-x} (vi) f(x)=$\frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

(vii) f(x)=$\frac{4\,sinx - 2x - x\,cosx}{2+cosx}$ (viii) f(x)= x^2-2log(2-x)-4x-7

Q-40 Find the values of x for which f(x)= $[x(x-2)]^2$ is an increasing function. also find the points on this curve at which the tangent is parallel to the x-axis.

Q-41 Prove that y=$\frac{4sin\theta}{2+cos\theta}$ - $\theta\ is$ an increasing function of θ ∈[0,$\pi/2$]

Q-42 Show that f(x) =$\tan^{-1}(sinx + cosx)$ is strictly increasing on x ∈ $\left(0, \frac{\pi}{2}\right)$

Q-43 Find all a ∈R for which the function f(x) = (a+2)$x^3 - 3ax^2$+9ax-1 decreasing function on x ∈R.

Q-44 Show that semi vertical angle of the cone of the maximum volume and given scant height is $tan^{-1}\sqrt{2}$,

Q-45 Show that the right circular cone of least curved surface and given volume has an altitude equal to$\sqrt{2}$ times the radius of base. **PAWAN**

Q-46 Show that right circular cylinder of give surface and $max.$ volume is such that its height is equal to the diameter of the base.

Q-47 It the length of three sides of a trapezium other that base are equal$to10cm.$ find the max area of trapezium .

Q-48A tank with rectangular base and rectangular sides, open at top is to be constructed so that its depth is $2m$ and volume is $8m^3$. if building of tank costs $Rs70$ per square maters for the base and Rs 45 per square meters for sides .what is the cost of least expensive tank.

Q-49 A point on the hypotenuse of a triangle is at distance a&b from the sides of triangle. Show that minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{3/2}$.

Q-50 Find the print on the curve $x^2 = 4y$ which is nearst to P (-1, 2),

Q-51 Show that the height of the cylinder of max volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. also find max volume.

Q-52 Show that the height of cylinder of max. Volume which can be inscribed in a circular cone of height h and semi vertical angle α is one third of the cone and max volume of cylinder is $\frac{4}{27}\pi h^3 tan^3\alpha$.

Q-53 Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is$\frac{8}{27}$ of the volume of sphere.

Q-54 A rectangle is inscribed in a semi circle of radius r with one of its side on diameter of semi -circle. Find the max. Area of this rectangle

Q-55 Show that all rectangles inscribed in a given fixed circle, the square has the maximum area.

Q-56 A jet of enemy is flying along the curve y=$x^2 + 2$ .a soldier is placed at the point (3,2) what is the nearest distance between the soldier and the jet.

Q-57 An open box with a square base, is to be made out of a given quantity of metal sheet of area c^2.show that the maximum volume of the box is$\frac{c^3}{6\sqrt{3}}$. **PAWAN**

Q-58 Show that the semi vertical angle of right circular cone of a given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

Q-59 The section of window is rectangle surmounted by an equilateral triangle. Given that its perimeter is 16 meteres.find the width of the window in order that maximum light may be admitted.

Q-60 Prove that the perimeter of a right angled triangle of the given hypotenuse is maximum when the triangle is isosceles.

Q-61 Show that the height of the right circular cylinder of maximum volume that can be inscribed in a cone of height H is H/3.

Q-62 Show that the triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

Q-63 A given quantity of metal is to be cast in to a half cylinder with a rectangle base and semi circular ends. Show that in order that the total surface area is minimum. The ratio of length of cylinder to the diameter of its semi circular ends is $\pi:(\pi+2)$.

Q-64 A window is the form of a rectangle surmounted by a semi circular opening. The total perimeter of window is 10 cm.find the dimensions of window to admit maximum light through the whole opening.

Q-65 If the sum of lengths of hypotenuse and a side of right-angled triangle is given. Show that the area of the triangle is maximum when the angle between them is 60^0.

Q-66 Show that the right circular cylinder, open at the top of given surface area and maximum volume is such that its height is equal to the radius of base.

Q-67 Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Q-68 At what point, the slope of the curve y=-$x^3+3x^2+9x-27$ is maximum. Also find the maximum slope.

Q-69 Prove that f(x) = sinx +$\sqrt{3}cosx$ has the maximum value at x=$\pi/6$.

Q-70 (i) Show that $\frac{logx}{x}$ has a maximum value at x=e. (ii) Find the absolute maximum and minimum value of the function

F(x) = $3x^4-8x^3+12x^2-48x+1$,x∈[1,4] **PAWAN**

Q-71 Find the local max. and local min value of f(x)=sinx-cosx ,$0<x<2\pi$

Q-72 Find the angle θ, lies in first quadrant, which increases twice as fast as its sine.

Q-73 A telephone company in a town has 500 subscribers on its list and collects the fixed charge of Rs 300/- per subscribers per year. The company proposes to increase the annual subscription and it is believed that for increases of each Rs1/-

one subscriber will discontinue the service. Find what increase will bring maximum profit.

Q-74 Show that the height of the cone of maximum volume that can be inscribed in a sphere of radius 12 cm is 16 cm.

Q-75An isosceles triangle of vertical angle $2\theta$is inscribed in a circle of radius a .show that the area of triangle is maximum when $\theta = \pi/3$.

Q-76 The curve y=a$x^3 + bx^2$+cx+5 touches the x axis at (-2,0) & cuts the y axis at the point Q where its gradient is 3.find the equation of curve completely.

Q-76 The cost of fuel for running a bus is proportional to the square of the speed generated in km/hr.it cost Rs 48 per hour when the bus is moving with a speed of 20 km/hr.what is the most economical speed if the fixed charges are Rs 108 for 1 hr,over and running charges.

Q-77 Find the angle of intersection of the curves xy=6 and x^2y=12.

Q-78 Find the values of x for which f(x) = x^x,x>0 is strictly increases and decreases.

Q-79 Does the straight line $\frac{x}{a} + \frac{y}{b} = 2$touches the curve $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$=2? If it touches then write the coordinates of the point of contact. **PAWAN**

Q-80 Find the equation of normal to the curve y=$(1 + x)^y + \sin^{-1}(sin^2 x)$ at x=0